Coloring square-free Berge graphs

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A *Berge* graph is a graph that contains no odd hole and no odd antihole of length at least 5.

Chudnovsky, Robertson, Seymour and Thomas proved in 2002 that every Berge graph is perfect (the "Strong Perfect Graph Theorem").

Grötschel, Lovász and Schrijver proved in 1984 that there exists a polynomial-time algorithm for determining the chromatic number of any perfect graph. However this algorithm is based on the ellipsoid method from linear programming and consequently is rather complex and opaque. It is still an open problem to find a "purely combinatorial" algorithm to compute the chromatic number of all perfect graphs. There are such algorithms for many special subclasses of perfect graphs, using various methods.

We will show here how to solve this problem for Berge graphs that do no contain a *square* (4-hole).

The algorithm works in two phases. When the graph does not contain a *prism* (a certain type of graph), then one can use the *even-pair* contraction technique developed in the 1990s by several authors. The conjecture, due to Everett and Reed, that this technique works for Berge graphs that contain no antihole and no prism was proved earlier by Trotignon and myself. When the graph contains a prism, we show that it admits a certain type of cutset, which yields a decomposition scheme for G. Then we show how optimal colorings of the leaves of the decomposition tree can be combined so as to produce an optimal coloring for the original graph.

This is joint work with Maria Chudnovsky, Irene Lo, Nicolas Trotignon and Kristina Vušković.